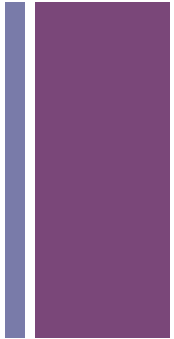


2.8.16-2.13.16



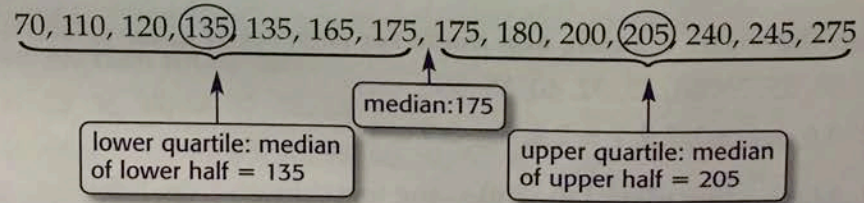
A median divides a data set into two parts. To divide the data into four parts, find the median of the lower half, called the **lower quartile (LQ)**, and the median of the upper half, called the **upper quartile (UQ)**. A **box-and-whisker plot** is a diagram that summarizes data by dividing it into four parts.

EXAMPLE Construct a Box-and-Whisker Plot

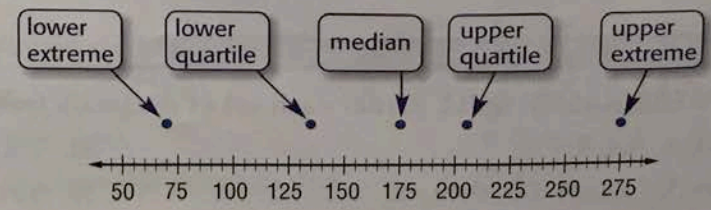
1 NUTRITION Make a box-and-whisker plot of the data in the table above.

Step 1 Order the data from least to greatest.

Step 2 Find the median and the quartiles.



Step 3 Draw a number line. The scale should include the median, the quartiles, and the least and greatest values, which are called the **lower extreme** and the **upper extreme**, respectively. Graph the values as points above the line.



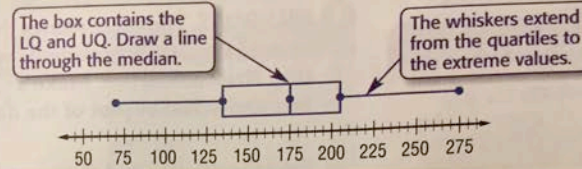
STUDY TIP

Length of Parts

The parts of a box-and-whisker plot are different lengths because of how the data are clustered or spread out.

- short part: data are clustered together
- long part: data are spread out

Step 4 Draw the box and whiskers.



Each of the four parts of a box-and-whisker plot contain one fourth of the data.

EXAMPLE Analyze Data

- 2 ARCHITECTURE** The table shows the heights of the ten tallest buildings in St. Louis, Missouri. Make a box-and-whisker plot of the data. Then use it to describe how the data are spread out.

Find the median, the quartiles, and the extremes. Then construct the plot.

$$\text{median} = \frac{485 + 420}{2} \text{ or } 452.5$$

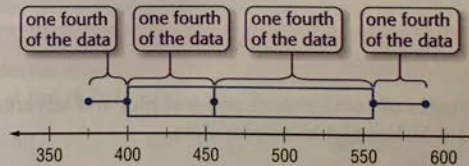
$$\text{LQ} = 398 \quad \text{UQ} = 557$$

$$\text{lower extreme} = 375$$

$$\text{upper extreme} = 593$$

Building	Height (ft)
Metropolitan Square Tower	593
One Bell Center	588
Thomas F. Eagleton Federal Courthouse	557
Mercantile Center Tower	540
Firstar Center	485
Boatmen's Plaza	420
Laclede Gas Building	400
SW Bell Telephone Building	398
Civil Courts Building	390
One City Center	375

Source: *The World Almanac*



The graph shows that half of the tallest buildings are between 398 and 557 feet high. The largest range of the four quartiles is from 452.5 to 557 feet. One-fourth of the tallest buildings are within these heights.

READING Math

Quartiles A *quartile* can also refer to any of the four parts of a box-and-whisker plot. So, each quartile contains one-fourth, or 25%, of the data.

Another way to describe how data are spread out is to use the **interquartile range**, or the difference between the upper quartile and the lower quartile.

In Lesson 2-3, you learned that outliers are values separated from the rest of the data. In a box-and-whisker plot, outliers are data that are more than 1.5 times the interquartile range from the quartiles. They are not included in the whiskers of a box-and-whisker plot.



Practice and Applications

SPORTS For Exercises 7–9, refer to the table at the right. It shows the regular season games won by each men's professional basketball team in a recent season.

Number of Wins					
25	36	46	15	30	53
40	32	17	45	41	31
56	50	52	47	26	48
43	56	51	50	55	58
44	47	53	23	19	

HOMEWORK HELP

For Exercises	See Examples
7, 8, 12	1
9, 13	2
10, 11	3

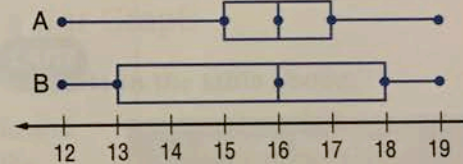
Extra Practice
See pages 568, 597.

- Find the lower extreme, LQ, median, UQ, and upper extreme.
- Draw a box-and-whisker plot of the data.
- What fraction of the data are between 45 and 51.5?

BRIDGES For Exercises 10–13, refer to table at the right. It shows the lengths of 22 of the longest suspension bridges in North America.

Length of Bridges (ft)			
4,260	2,310	1,800	1,595
4,200	2,300	1,750	1,550
3,800	2,190	1,632	1,500
3,500	2,150	1,600	1,500
2,800	2,000	1,600	
2,800	1,850	1,600	

- Determine the interquartile range.
- What are the limits on outliers? Are there any outliers?
- Draw a box-and-whisker plot of the data.
- What fraction of the bridges are between 1,600 and 2,800 feet long?
- Compare and contrast the data represented in the box-and-whisker plots at the right.
- CRITICAL THINKING** Describe a set of data in which there is only one whisker in its box-and-whisker plot.





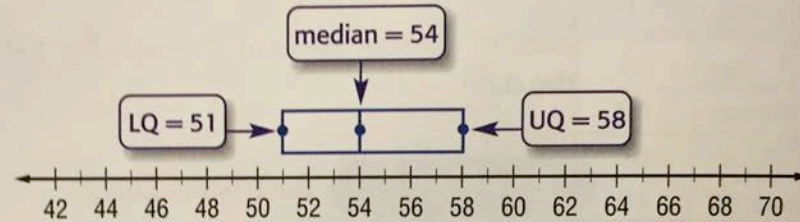
EXAMPLE

Identify and Plot Outliers

- 3 PRESIDENTS** The table shows the ages of the last 21 presidents when they first took office. Make a box-and-whisker plot of the data.

Ages of Presidents						
47	55	54	42	51	56	55
51	54	51	60	62	43	55
56	61	52	69	64	46	54

Find the median and the quartiles. Plot the points and draw a box.



Next, determine whether there are any outliers.

interquartile range: $UQ - LQ = 58 - 51$ or 7

So, outliers are data more than $1.5(7)$ or 10.5 from the quartile.

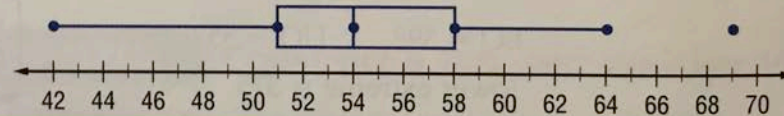
lower limit: $LQ - 10.5 = 51 - 10.5$ or 40.5

upper limit: $UQ + 10.5 = 58 + 10.5$ or 68.5

40.5 and 68.5 are limits for the outliers.

Any data point that is less than 40.5 or greater than 68.5 is an outlier.

So, 69 is an outlier. Plot the outlier using a dot. Then draw the lower whisker to the lower extreme, 42, and the upper whisker to the greatest value that is not an outlier, 64.



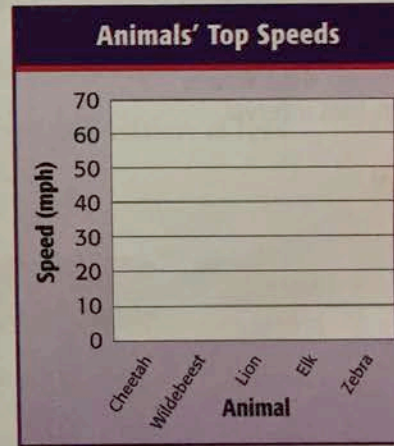


A **bar graph** is one method of comparing data by using solid bars to represent quantities.

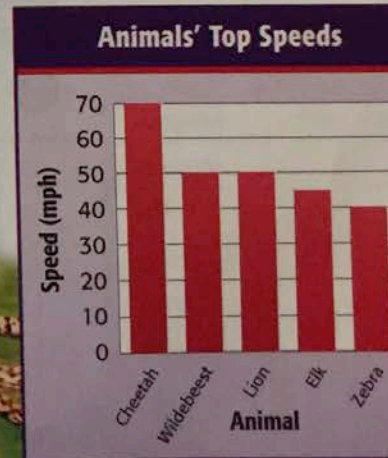
EXAMPLE Construct a Bar Graph

1 Make a bar graph to display the data in the table above.

Step 1 Draw a horizontal axis and a vertical axis. Label the axes as shown. In this case, the scale on the vertical axis is chosen so that it includes all the speeds. Add a title.



Step 2 Draw a bar to represent each category. In this case, a bar is used to represent the speed of each animal.



A special kind of bar graph, called a **histogram**, uses bars to represent the frequency of numerical data that have been organized in intervals.

EXAMPLE Construct a Histogram

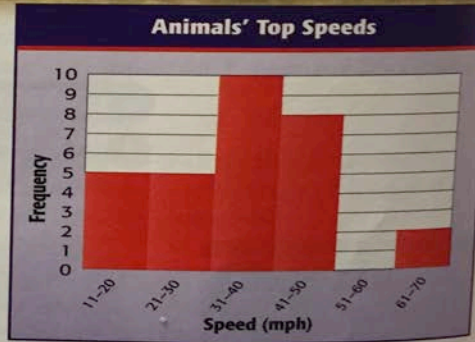
- 2 ANIMALS** The top speeds of thirty different land animals have been organized into a frequency table. Make a histogram of the data.

Maximum Speed (mph)	Frequency
11–20	5
21–30	5
31–40	10
41–50	8
51–60	0
61–70	2



Source: *The World Almanac*

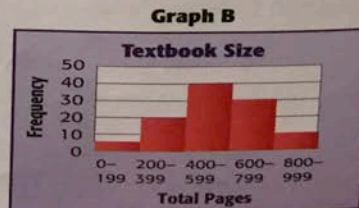
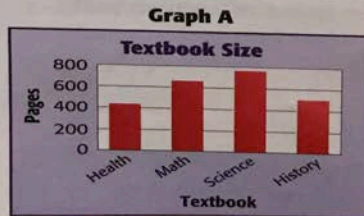
- Step 1** Draw and label horizontal and vertical axes. Add a title.
- Step 2** Draw a bar to represent the frequency of each interval.



The two highest bars represent a majority of the data. From the graph, you can easily see that most of the animals have a top speed of 31–50 miles per hour.

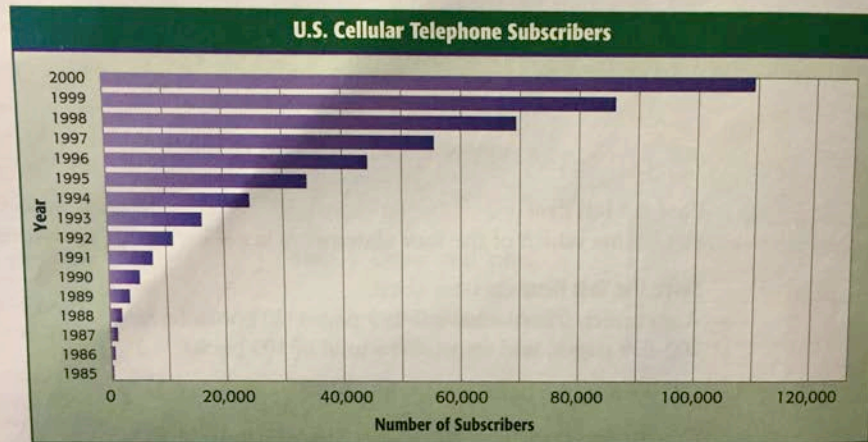
EXAMPLE Compare Bar Graphs and Histograms

- 3 TEXTBOOKS** Refer to the graphs below.





TECHNOLOGY For Exercises 11–13, use the bar graph below.



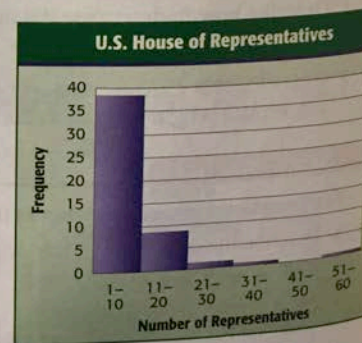
Source: *The World Almanac*

- How is this bar graph different from the other bar graphs in the lesson?
- Describe how the number of cell phone subscribers grew over the 15-year period.
- Use the graph to predict the number of cell phone subscribers in 2006.

CIVICS For Exercises 14–16, use the histogram at the right and the information below.

Congress is made up of senators and representatives. Each state has 2 senators, and the number of representatives per state is determined by the state's population.

- What does each interval represent?
- Describe general patterns in the histogram.
- There are 435 members in the House of Representatives. Determine the mean number of representatives per state. Compare this value to the value of the median number, according to the histogram. Discuss the significance of 38 states having 10 or fewer representatives.



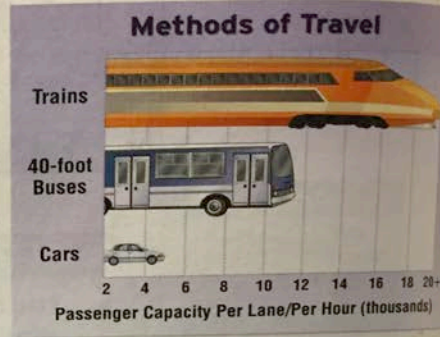


Misleading Statistics

WHEN am I ever going to use this?

TRANSPORTATION A graph like the one at the right appeared in a brochure describing various modes of transportation.

1. Approximately how many more passengers per lane can a 40-foot bus transport in an hour than a car can transport?
2. Is the bus on the graph about twice as large as the car? Explain.
3. Do you think the graph appeared in a brochure for a train/bus transit company or for a car company? What makes you think so?

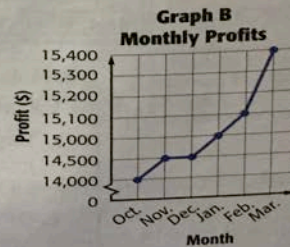
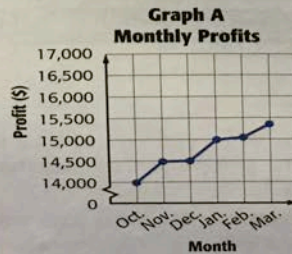


The graph above shows that a car can transport about 5,000 people per lane, per hour. A train can transport about 4 times that many. However, the train on the graph is much more than 4 times larger than the car. For this reason, the graph could be considered misleading.

Other ways that graphs could be misleading are if there is no title, there are no labels on the axes or scales, or the intervals on a scale are not equal.

EXAMPLE Misleading Graphs

1 BUSINESS The line graphs below show monthly profits of a company from October to March. Which graph is misleading? Explain.



Graph B is misleading because the vertical scale has intervals of both \$500 and \$100. The increase in profits are exaggerated.

Statistical averages used to describe data may also be misleading. For example, whenever there are outliers, the mean may not be a good way to describe a data set because it is affected the most by outliers.

EXAMPLE

Misleading Statistics

- 2 SALARY** ABC Corporation claims the average salary for its employees is more than \$60,000, while the average salary at XYZ Incorporated is only \$25,000. Use the table to explain their reasoning and determine where you would prefer to work.

Position	Salary (\$)	
	ABC Corp.	XYZ Inc.
President	500,000	120,000
1st Vice President	400,000	85,000
2nd Vice President	240,000	75,000
Sales Staff (5)	20,000	40,000
Supporting Staff (8)	15,000	25,000
Catalog Staff (7)	9,000	22,500

	<u>ABC Corp.</u>	<u>XYZ Inc.</u>
mean salary	\$61,870	\$36,413
median salary	\$15,000	\$25,000

ABC Corporation used the mean to represent its average salary. They used the median to represent the average salary at XYZ Inc. Unless you could be a president or vice president, it would be better to work for XYZ Inc. because its 20 lowest-paid employees are all better paid than any of the 20 lowest-paid employees at ABC Corporation.



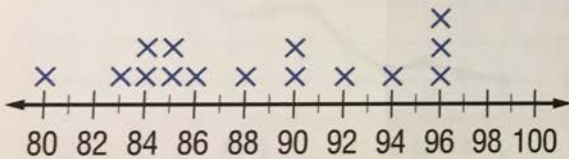
GEOGRAPHY For Exercises 9–11, use the table at the right.

9. Find the mean, mode, and median of the data.
10. Which measure of central tendency would be misleading in describing the size of the islands? Explain.
11. Which measure would most accurately describe the data?

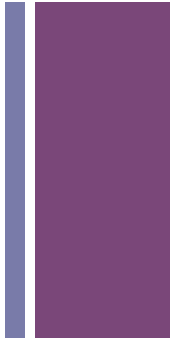
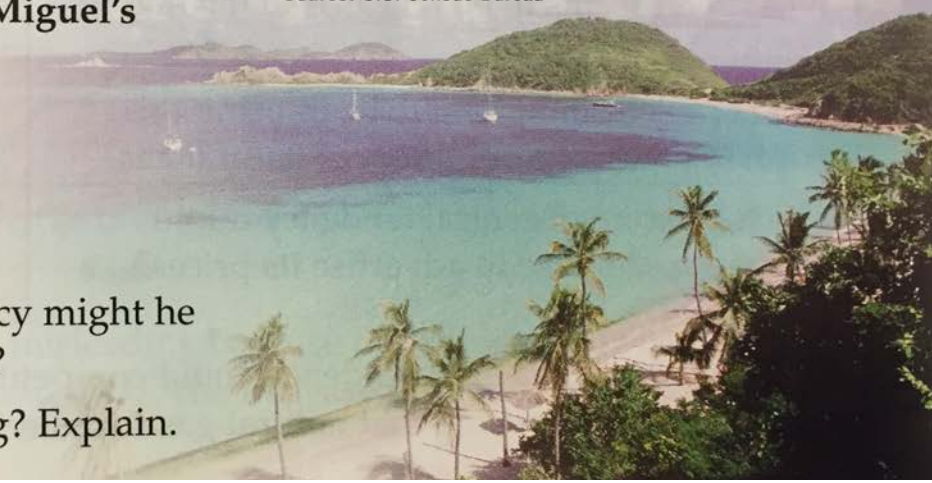
Caribbean Islands			
Island	Area (sq mi)	Island	Area (sq mi)
Antigua	108	Martinique	425
Aruba	75	Puerto Rico	3,339
Barbados	166	Tobago	116
Curacao	171	Virgin Islands, UL	59
Dominica	290	Virgin Islands U.S.	134

Source: U.S. Census Bureau

TESTS For Exercises 12 and 13, use Miguel's test scores shown in the line plot.

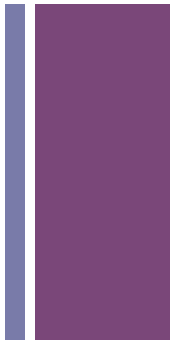


12. Which measure of central tendency might he use to emphasize the high scores?
13. Would this number be misleading? Explain.





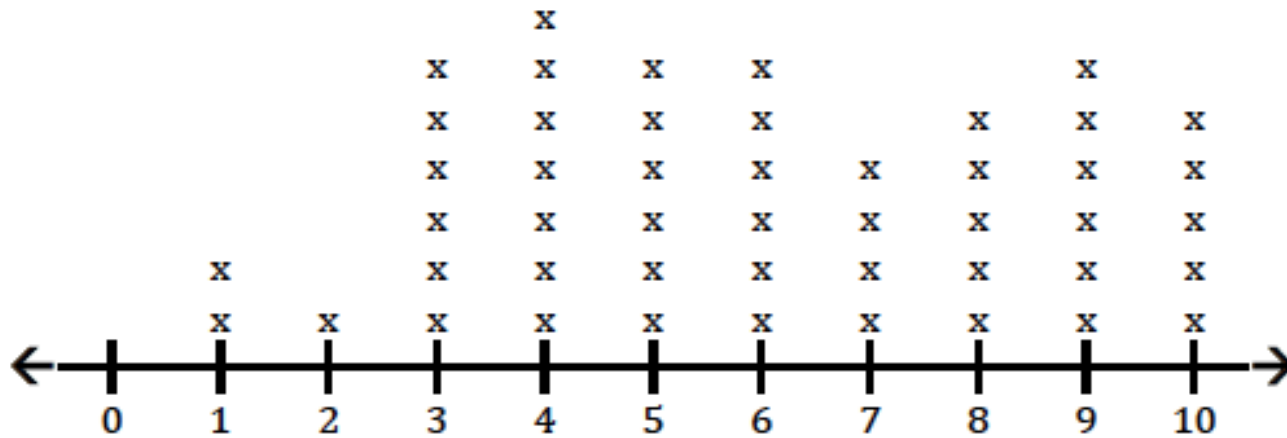
stem	leaf
13	6
14	1 1 4 6
15	3 8
16	5 8
17	2 3 6
18	0 6 7
19	
20	5
21	1 8
22	0 2 6 8



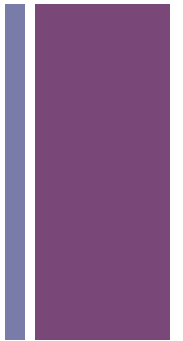
1. Determine the minimum value, maximum value and range of the data.
2. Determine the count, median, mode and mean of the data. Round the mean to one decimal place if necessary.
3. How many values are greater than 190?
4. How many values are less than 151?

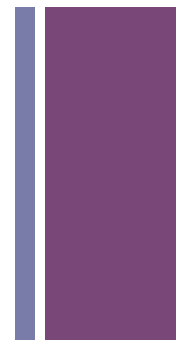


Line Plot 1



1. Determine the minimum value, maximum value and range of the data.
2. Determine the count, median, mode and mean of the data. Round the mean to one decimal place if necessary.
3. How many values are greater than 10?
4. How many values are less than 10?





Test scores	frequency
65-69	3
70-74	5
75-79	2
80-84	8
85-89	12
90-94	15
95-100	9

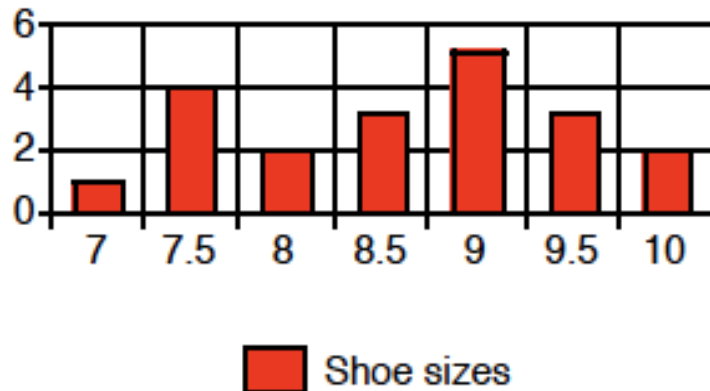
How many students were tested? _____

What is the mean test score? _____

What is the median test score? _____

What test score is the mode? _____

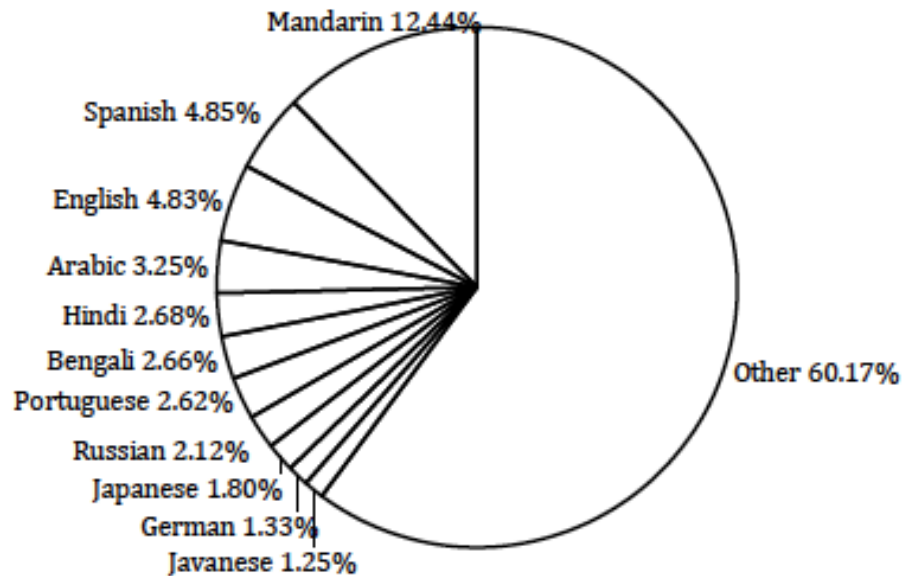
Shoe Sizes



1. What is the range of shoe sizes? _____
2. What is the mode of show sizes? _____
3. What is the mean shoe size? _____
4. What is the median shoe size? _____

Languages of the World (2009 Estimate)

Percentage of people who speak each language as their first language.



Source of data: <https://www.cia.gov/library/publications/the-world-factbook/geos/xx.html>

Which language is spoken by the most people?

What percentage of people speak one of the top 10 languages?

What percentage of people speak one of the top 5 languages?

If the world population was 6.8 billion in 2009, how many of those people spoke Portuguese?

There are an estimated 7,100 languages spoken in the world. Do you think this number will increase or decrease? Explain your answer.

ANALYZE DISPLAYS Find the mean, median, mode, and range of the data represented. Round to the nearest tenth if necessary.

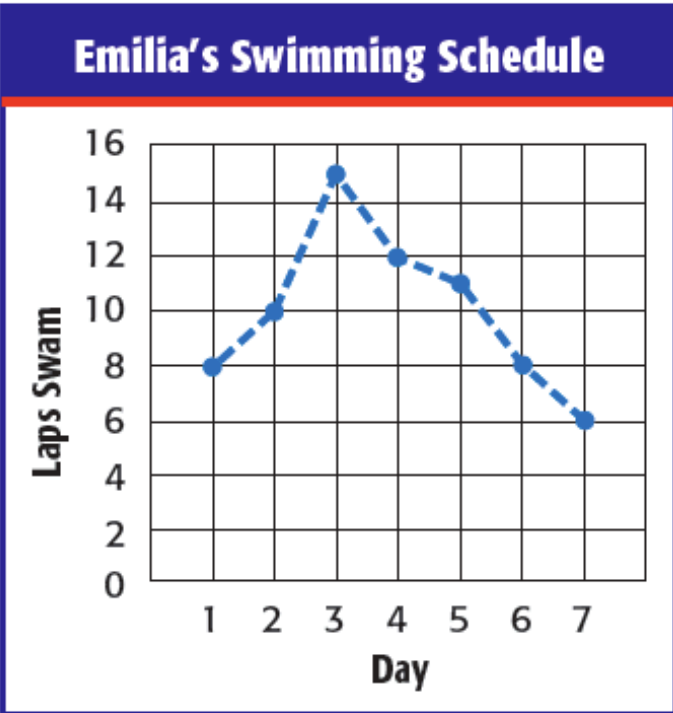
9.

Average Speeds (mph)			
40	52	44	46
52	40	44	50
41	44	44	50

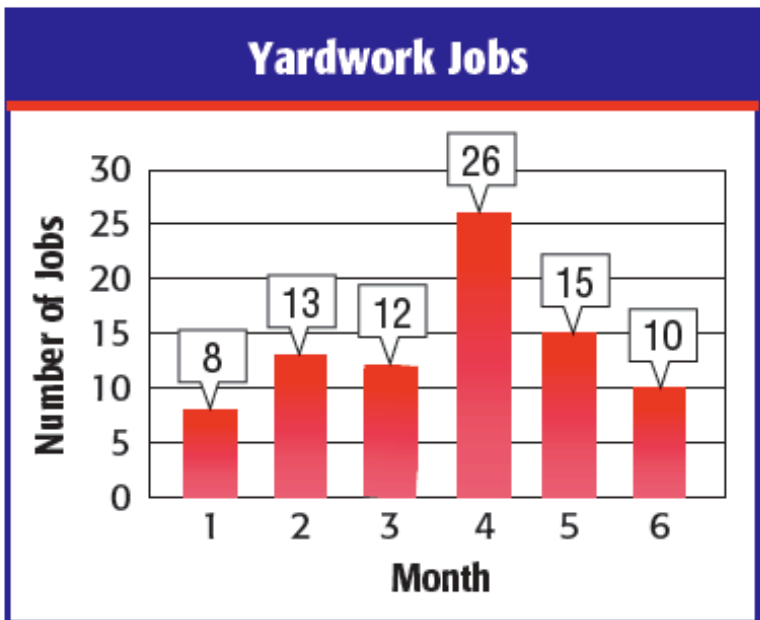
10.

Test Grades			
100	77	80	65
87	85	85	82
100	97	95	75

11.

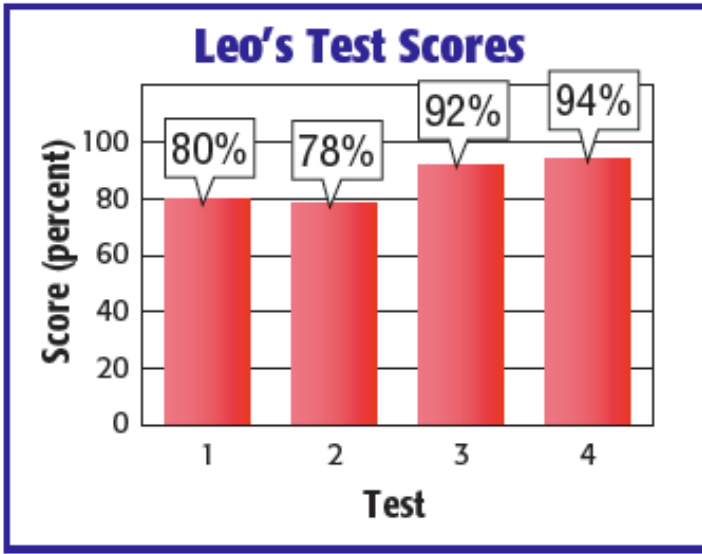
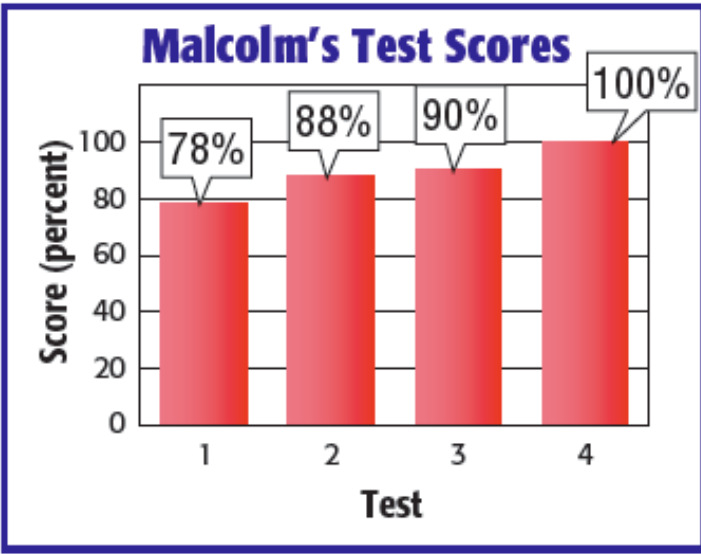


12.





Malcolm's and Leo's last four math test scores are shown in the graphs below. Write a sentence comparing the mean scores of the students. (Lesson 1B)



The 48 pieces of taffy in the box above are called **outcomes**. A **simple event** is one outcome or a collection of outcomes. For example, picking a piece of vanilla taffy is a simple event. The chance of that event happening is called **probability**.

Key Concept

Probability

Words The probability of an event is a ratio that compares the number of favorable outcomes to the number of possible outcomes.

Symbols $P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$

Outcomes occur at **random** if each outcome occurs by chance. A piece of vanilla taffy was selected at random in the activity above.

Read $P(\text{vanilla})$ as the probability of picking a piece of vanilla taffy.

$$P(\text{vanilla}) = \frac{\text{number of pieces of vanilla taffy}}{\text{total number of pieces of taffy}}$$

$= \frac{1}{8}$, or 12.5% Simplify.

EXAMPLE Find Probability

- 1 What is the probability of rolling an even number on a number cube marked with 1, 2, 3, 4, 5, and 6 on its faces?

$$\begin{aligned} P(\text{even number}) &= \frac{\text{even numbers possible}}{\text{total numbers possible}} \\ &= \frac{3}{6} \quad \text{Three numbers are even: 2, 4, and 6.} \\ &= \frac{1}{2} \quad \text{Simplify.} \end{aligned}$$



The probability of rolling an even number is $\frac{1}{2}$ or 50%.



EXAMPLE Find Probability

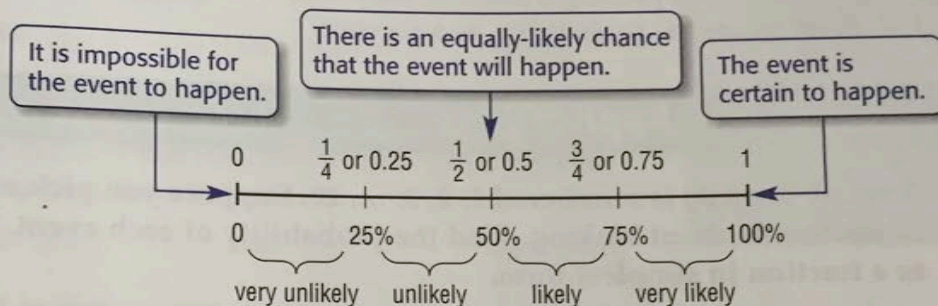
2 PARCHEESI Jewel rolls two number cubes. She can move her game piece if a total of 5 is shown on the number cubes, or if a 5 is shown on at least one number cube. What is the probability that she can move her game piece on one roll of the number cubes?

List all the possible outcomes. Then, find the pairs that total 5, or if a 5 is shown on at least one of the number cubes.

1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

There are 36 possible outcomes and 15 of them are favorable. So, the probability that Jewel moves her game piece in one roll is $\frac{15}{36}$, or $\frac{5}{12}$.

The probability that an event will happen is somewhere between 0 and 1. It can be shown on a number line.



Either Jewel will be able to move her game piece or she will *not* be able to move it. Two events that are the only ones that can possibly happen are examples of **complementary events**.

EXAMPLE Find a Complementary Event

- 3 PARCHEESI** Refer to Example 2. Find the probability that Jewel *cannot* move her game piece on one roll of the number cubes.

$$P(A) + P(\text{not } A) = 1$$

$$\frac{5}{12} + P(\text{not } A) = 1 \quad \text{Substitute } \frac{5}{12} \text{ for } P(A).$$

$$\begin{array}{r} \frac{5}{12} + P(\text{not } A) = 1 \\ - \frac{5}{12} \qquad \qquad - \frac{5}{12} \\ \hline P(\text{not } A) = \frac{7}{12} \end{array} \quad \text{Subtract } \frac{5}{12} \text{ from each side.}$$

$$P(\text{not } A) = \frac{7}{12} \quad \text{Simplify.}$$

So, the probability that Jewel *cannot* move her game piece is $\frac{7}{12}$.

Practice and Applications

A set of 20 cards is numbered 1, 2, 3, ..., 20. Suppose you pick a card at random without looking. Find the probability of each event. Write as a fraction in simplest form.

10. $P(1)$
11. $P(\text{not a factor of } 10)$
12. $P(\text{multiple of } 3)$
13. $P(\text{even number})$
14. $P(\text{less than or equal to } 20)$
15. $P(3 \text{ or } 13)$
16. How likely is it that an event with a probability of 0.28 will occur?
17. The forecast for tomorrow says that there is a 37% chance of rain. Describe the complementary event and its probability.

STUDENT COUNCIL The table shows the members of the Student Council. Suppose one student is randomly selected as the president. Find the probability of each event. Write as a fraction in simplest form.

18. $P(\text{girl})$
19. $P(\text{not } 7\text{th grader})$
20. $P(\text{boy})$
21. $P(8\text{th grader})$
22. $P(\text{boy or girl})$
23. $P(6\text{th or } 8\text{th grader})$
24. $P(\text{not } 6\text{th grader})$
25. $P(5\text{th grader})$
26. Which event has a greater chance of happening: picking a president who is a girl or a president who is not an 8th grader? Explain.

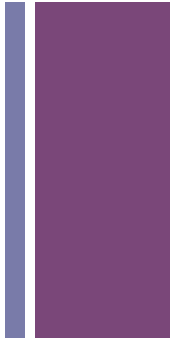
HOMEWORK HELP

For Exercises	See Examples
10–15	1
18, 20–23, 25–27	2
17, 19, 24, 28–29	3

Extra Practice
See pages 584, 604.

Student Council

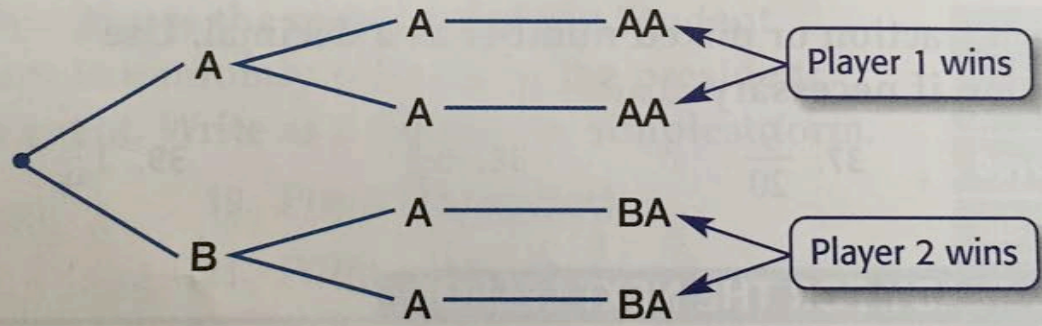
girls	30
boys	20
8th graders	25
7th graders	15
6th graders	10



A game in which players of equal skill have an equal chance of winning is a **fair game**. One way you can analyze whether games are fair is by drawing a **tree diagram**. A tree diagram is used to show all of the possible outcomes, or **sample space**, in a probability experiment.

EXAMPLE Draw a Tree Diagram

1 GAMES Refer to the Mini Lab above. Draw a tree diagram to show the sample space. Then determine whether the game is fair.



There are four equally-likely outcomes with two favoring each player. So, the probability that each player can win is $\frac{1}{2}$. Thus, the game is fair.

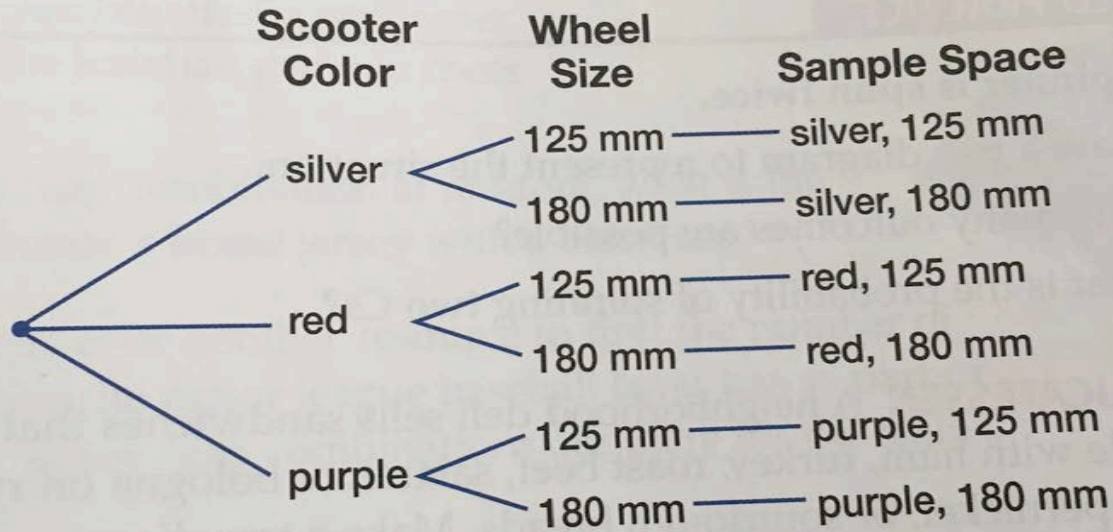


EXAMPLE

Find the Number of Outcomes

2 SCOOTERS A certain type of kickboard scooter comes in silver, red, or purple with wheel sizes of 125 millimeters or 180 millimeters. Find the total number of color-wheel size combinations.

Make a tree diagram to show the sample space.



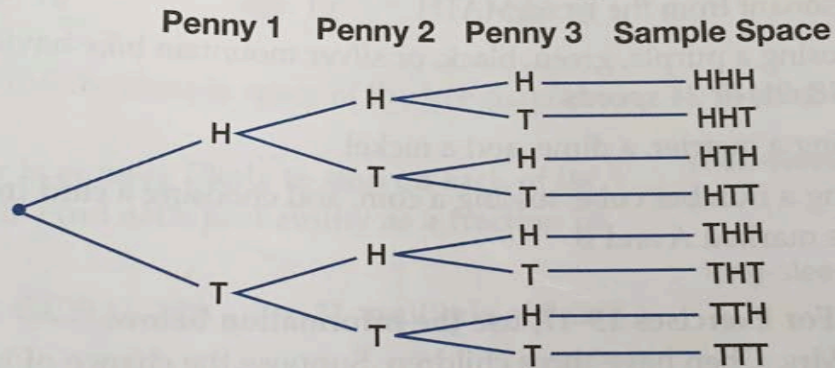
There are six different color and wheel size combinations.

EXAMPLE

Find Probability Using Tree Diagrams

- 3 COINS** Suppose Pablo tosses three pennies. Find the probability that all three will show heads.

Make a tree diagram to show the sample space. Then, find the probability of the three pennies showing heads.



The sample space contains 8 possible outcomes. Only 1 outcome has all pennies showing heads. So, the probability of three pennies showing heads is $\frac{1}{8}$, or 12.5%.

Your Turn Find each probability.

- b. $P(3 \text{ tails})$ c. $P(\text{exactly } 2 \text{ heads})$ d. $P(\text{at least } 1 \text{ tail})$



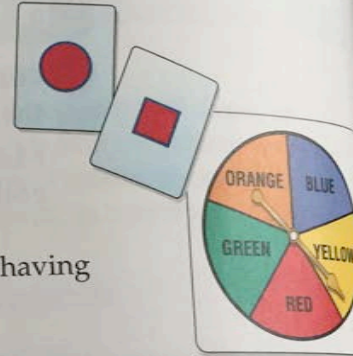
For each situation, make a tree diagram to show the sample space. Then give the total number of outcomes.

- tossing a coin and rolling a number cube
- choosing black, blue, or brown socks with boots, gym shoes, or dress shoes
- picking a number from 1 to 5 and choosing the color red, white, or blue
- choosing a card with a shape and spinning the spinner from the choices at the right
- choosing a letter from the word SPACE and choosing a consonant from the word MATH
- choosing a purple, green, black, or silver mountain bike having 10, 18, 21, or 24 speeds
- tossing a quarter, a dime, and a nickel
- rolling a number cube, tossing a coin, and choosing a card from the cards marked A and B

WORK HELP

For Exercises See Examples
7–15, 18, 21 1, 2
16–17, 19 3

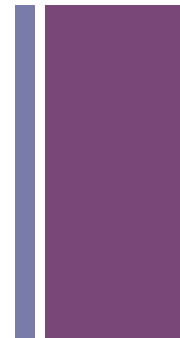
Extra Practice
See pages 585, 604.



FAMILY For Exercises 15–17, use the information below.

Mr. and Mrs. Chen have three children. Suppose the chance of having either a boy or a girl is 50%.

- Draw a tree diagram showing the possible arrangements of gender of the three children.
- What is the probability of having 2 boys and 1 girl?
- What is the probability of having the first two children boys and the last child a girl?



In the activity above, you discovered that multiplication, instead of a tree diagram, can be used to find the number of possible outcomes in a sample space. This is called the **Fundamental Counting Principle**.

Key Concept

Fundamental Counting Principle

If event M can occur in m ways and is followed by event N that can occur in n ways, then the event M followed by N can occur in $m \times n$ ways.

EXAMPLE

Use the Fundamental Counting Principle

1 FOOD A famous steak house allows customers to create their own steak dinners. The choices are shown at the right. How many different steak dinners are possible?

451 Steakhouse		
Steak	How Steaks Are Cooked	Potatoes
New York Strip	rare	mashed
Ribeye	medium	baked
Filet	well	twice baked
Porterhouse		au gratin
T-Bone		

$$\underbrace{\text{types of steaks}}_5 \cdot \underbrace{\text{number of ways steaks can be cooked}}_3 \cdot \underbrace{\text{types of potatoes}}_4 = \underbrace{\text{total number of steak dinners}}_{60}$$

There are 60 different ways of choosing a steak dinner.

Check You can check your work by drawing a tree diagram and listing the 60 outcomes.



Use the Fundamental Counting Principle to find the total number of outcomes in each situation.

9. rolling a number cube and tossing two coins
10. choosing a number from 1 to 20 and a color from 7 colors
11. choosing a plain, blueberry, garlic, or cinnamon-and-raisin bagel, with plain, chive, or sun-dried tomato cream cheese
12. choosing iced tea in regular, raspberry, lemon, or peach flavors; sweetened or unsweetened; and in a glass or in a plastic container
13. picking a date in the month of May and a day of the week
14. choosing a 4-digit Personal Identification Number (PIN) if the digits cannot be repeated
15. **ADVERTISING** The Wake-Up Restaurant advertises that you can have a different pancake breakfast every day of the year. It offers 25 different kinds of pancakes and 14 flavored syrups. If the restaurant is open every day of the year, is its claim valid? Explain.
16. **GAMES** A player rolls five number cubes to score the maximum number of points. Find the number of outcomes possible in one roll.

CRITICAL THINKING How many outcomes are possible if you toss each of the following?

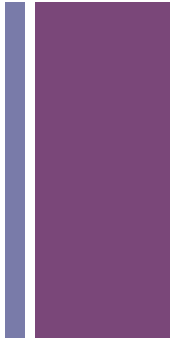
17. one coin

18. two coins

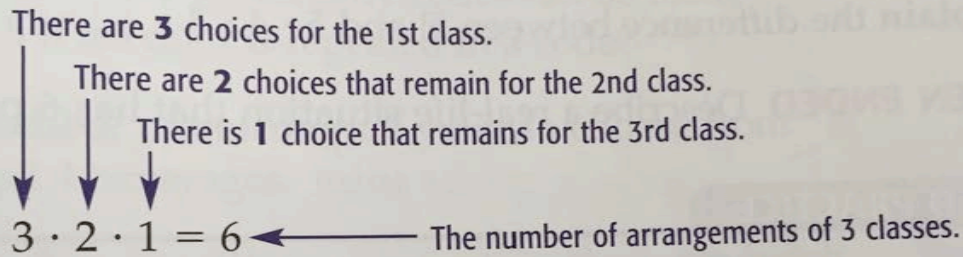
19. three coins

20. n coins

HOMEWORK HELP	
For Exercises	See Example
9–14	1
15–16	2
Extra Practice See pages 585, 6	



A **permutation** is an arrangement, or listing, of objects in which order is important. You can use the Fundamental Counting Principle to find the number of possible arrangements.



The expression $3 \cdot 2 \cdot 1$ can be written as $3!$, which is read *three factorial*.

Key Concept

Factorial

The expression n **factorial** ($n!$) is the product of all counting numbers beginning with n and counting backward to 1.



Find the value of each expression.

7. $5!$ 8. $9!$ 9. $4! \cdot 3!$ 10. four factorial
11. $3! \cdot 6!$ 12. $10 \cdot 9 \cdot 8$ 13. $5! \cdot 4!$ 14. $8! \cdot 2!$

15. In how many ways can a softball manager arrange the first four batters in a lineup of nine players?
16. How many different 5-digit zip codes are there if no digit is repeated?
17. **MUSIC** The chromatic scale has 12 notes. In how many ways can a song start with 4 different notes from that scale?

DOGS For Exercises 18 and 19, use the information below and at the right.

During the annual Westminster Dog Show, the best dog in each breed competes to win one of four top ribbons in the group.

18. In how many ways can a ribbon be awarded to a breed of dog in the Working group?
19. The top dog in each group competes against the other six group winners for Best of Show. If each dog has an equally-likely chance of winning Best of Show, what is the probability that a terrier will win?

For Exercises	See
7-14	
16	
15, 17-18	
Extra P	
See pages	

2002 Westm



Group
Herdin
Hound
Non-s
Sport