Probability

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A **permutation** is an arrangement, or listing, of objects in which order is important. You can use the Fundamental Counting Principle to find the number of possible arrangements.

There are **3** choices for the 1st class.

There are **2** choices that remain for the 2nd class.

There is **1** choice that remains for the 3rd class.

$3 \cdot 2 \cdot 1 = 6$ ← The number of arrangements of 3 classes.

The expression $3 \cdot 2 \cdot 1$ can be written as 3!, which is read *three factorial.*

Key Concept — Factorial

The expression *n* **factorial** (*n*!) is the product of all counting numbers beginning with *n* and counting backward to 1.

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EXAMPLES Evaluate Factorials

Find the value of each expression.

1 **7!**

$7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ Definition of factorial

$= 5{,}040$ Simplify.

2 **$2! \cdot 3!$**

$2! \cdot 3! = 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1$ Definition of factorial

$= 12$ Simplify.

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EXAMPLE Find a Permutation

3 **VOLLEYBALL** **In how many ways can the starting six players of a volleyball team stand in a row for a picture?**

This is a permutation that can be written as 6!.

$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ Definition of factorial

$= 720$ Simplify.

So, there are 720 ways the six starting players can stand in a row.

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EXAMPLE **Find a Permutation**

4 **SWIMMING** **The finals of the Middle School Appalachian League features 8 swimmers. In how many ways can the swimmers finish in first or second place?**

There are 8 choices for first place and 7 choices that remain for second place. So, there are $8 \cdot 7$ or 56 choices for first and second place.

Practice and Applications

Find the value of each expression.

7. 5!
8. 9!
9. $4! \cdot 3!$
10. four factorial
11. $3! \cdot 6!$
12. $10 \cdot 9 \cdot 8$
13. $5! \cdot 4!$
14. $8! \cdot 2!$

HOMEWORK HELP

For Exercises	See Examples
7–14	1, 2
16	3
15, 17–18	4

Extra Practice
See pages 585, 604.

15. In how many ways can a softball manager arrange the first four batters in a lineup of nine players?

16. How many different 5-digit zip codes are there if no digit is repeated?

17. **MUSIC** The chromatic scale has 12 notes. In how many ways can a song start with 4 different notes from that scale?

DOGS For Exercises 18 and 19, use the information below and at the right.

During the annual Westminster Dog Show, the best dog in each breed competes to win one of four top ribbons in the group.

18. In how many ways can a ribbon be awarded to a breed of dog in the Working group?

19. The top dog in each group competes against the other six group winners for Best of Show. If each dog has an equally-likely chance of winning Best of Show, what is the probability that a terrier will win?

2002 Westminster Dog Show

Group	Number of Breeds
Herding	19
Hounds	25
Non-sporting	18
Sporting	27
Terriers	27
Toy	22

Data Update Which breed of dog has won the most Best of Shows?

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An arrangement, or listing, of objects in which order is *not* important is called a **combination**. For example, in the activity above, choosing Alita and Bailey is the same as choosing Bailey and Alita.

Permutations and combinations are related. You can find the number of combinations of objects by dividing the number of permutations of the entire set by the number of ways each smaller set can be arranged.

A permutation of 4 players, taken 2 at a time.

$$\frac{4 \cdot 3}{2!} = \frac{4 \cdot 3}{2 \cdot 1} = \frac{12}{2} = 6$$

There are 2! ways to arrange 2 players.

EXAMPLE Find the Number of Combinations

1 **FOOD** Paul's Pizza Parlor is offering a large two-topping pizza for $14.99. There are five toppings from which to choose. How many different two-topping pizzas are possible?

Method 1 Make a list.

The five toppings are labeled pepperoni (p), sausage (s), onions (o), mushrooms (m), and green pepper (g).

p, s	p, m	p, o	p, g	s, m
s, o	s, g	m, o	m, g	o, g

Method 2 Use a permutation.

There are 5 · 4 permutations of two toppings chosen from five.

There are 2! ways to arrange the two toppings.

$$\frac{5 \cdot 4}{2!} = \frac{20}{2} = 10$$

So, there are 10 different two-topping pizzas.

EXAMPLE **Use a Combination to Solve a Problem**

2 **CHECKERS** A checkers tournament features each of the top 8 regional players playing every opponent one time. The 2 players with the best records will then play in a final round to determine the champion. How many matches will be played if there are no ties?

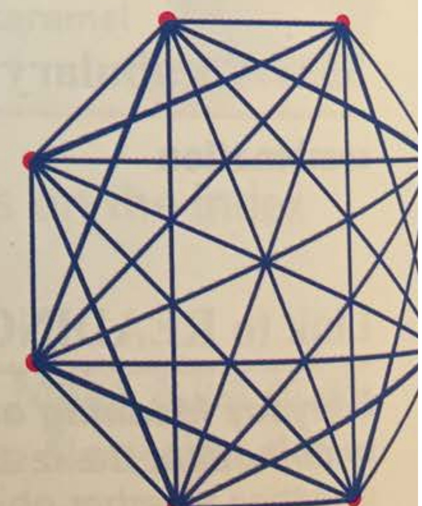
Find the number of ways 2 players can be chosen from a group of 8.

There are 8 · 7 ways to choose 2 people. →
There are 2! ways to arrange 2 people. →

$$\frac{8 \cdot 7}{2!} = \frac{56}{2} = 28$$

There are 28 matches plus 1 final match to determine the champion. So, there will be 29 matches played.

Check Make a diagram in which each person is represented by a point. Draw line segments between two points to represent the games. There are 28 line segments. Then add the final-round match to make a total of 29 matches.



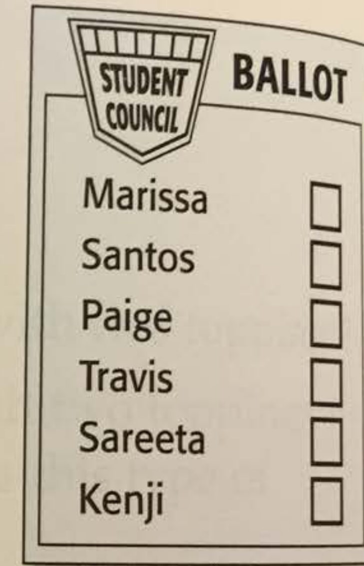
Your Turn

a. How many matches will be played if the top 16 players were invited to play?

EXAMPLES **Identify Permutations and Combinations**

Tell whether each situation represents a *permutation* or *combination*. Then solve the problem.

3 **STUDENT GOVERNMENT** **The six students listed at the right are members of Student Council. How many ways can you choose a president, vice president, and treasurer from this group?**



This is a permutation because the order of president, vice president, and treasurer is important. So, the number of ways you can choose the three officers is $6 \cdot 5 \cdot 4$, or 120 ways.

4 **In how many ways can you choose a committee of three students from the six members in student council shown above?**

This is a combination because the order of the students in the committee is not important.

There are $6 \cdot 5 \cdot 4$ ways to choose 3 people. →
There are 3! ways to arrange 3 people. →

$$\frac{6 \cdot 5 \cdot 4}{3!} = \frac{120}{6} = 20$$

So, there are 20 ways to choose the committee.

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7. **FOOD** The International Club is selling hot dogs at the Spring Carnival. Customers can select three toppings from among chili, onions, cheese, mustard, or relish. How many combinations of three-topping hot dogs are there?

HOMEWORK HELP

For Exercises	See Examples
7–10	1, 2
11–16	3, 4

Extra Practice
See pages 586, 604.

CIVICS For Exercises 8 and 9, use the information below and at the right.
If five of the nine Justices on the 2002 United States Supreme Court agree on a decision, they can issue a majority opinion.

8. How many different combinations of five Supreme Court Justices are there?

9. Before they take the Bench each day, the Justices engage in the "Conference handshake." Each Justice shakes hands with each of the other eight. How many handshakes take place?

U.S. Supreme Court, 2002	
Chief Justice, William H. Rehnquist	
Stephan G. Breyer	Antonin Scalia
Ruth Bader Ginsburg	David H. Souter
Anthony M. Kennedy	John Paul Stevens
Sandra Day O'Connor	Clarence Thomas

Source: www.washingtonpost.com

10. **SOCCER** There are 21 players trying out for 15 spots on the soccer team. How many ways does the coach have to create her team?

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Tell whether each problem represents a *permutation* or *combination*. Then solve the problem.

11. How many ways can you select four essay questions out of a total of 10 on the exam?
12. Six children remain in a game of musical chairs. If two chairs are removed, how many different groups of four students can remain?
13. How many ways can three flute players be seated in the first, second, or third seats in the orchestra?
14. In how many ways can four paintings be displayed from a collection of 15?
15. How many ways can seven students line up to buy concert tickets?
16. Given 12 Web sites, how many ways can you visit half of them?
17. **WRITE A PROBLEM** Write about a real-life situation that can be solved using a permutation and one that can be solved using a combination. Then solve both problems.
18. **CRITICAL THINKING** At a party, there were 105 handshakes. If each person shook hands exactly once with every other person, how many people were at the party?

In activity above, you found the experimental probability of rolling a sum of 7 on two number cubes. **Experimental probability** is found using frequencies obtained in an experiment or game.

The expected probability of an event occurring is called **theoretical probability**. This is the probability that you have been using since Lesson 9-1. The theoretical probability of rolling a sum of 7 on two number cubes is $\frac{6}{36}$, or $\frac{1}{6}$.

EXAMPLE Experimental Probability

1 **Two number cubes are rolled seventy-five times and a sum of 9 is rolled ten times.**

What is the experimental probability of rolling a sum of 9?

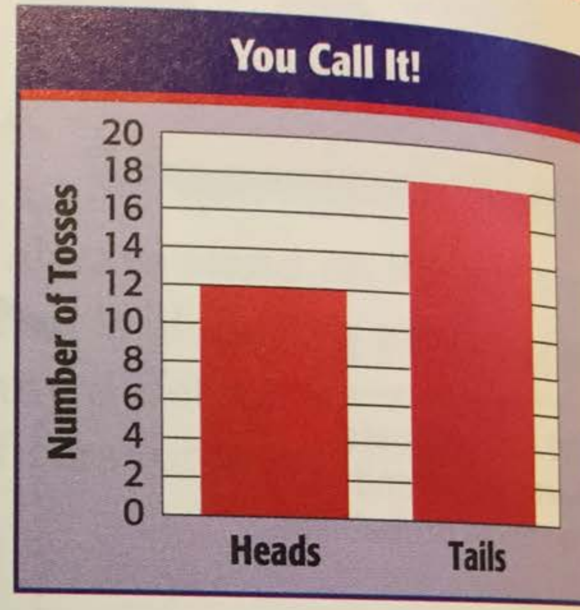
$$P(9) = \frac{\text{number of times a sum of 9 occurs}}{\text{number of possible outcomes}}$$

$$= \frac{10}{75} \text{ or } \frac{2}{15}$$

The experimental probability of rolling a sum of 9 is $\frac{2}{15}$.

EXAMPLES Experimental and Theoretical Probability

2 **The graph shows the results of an experiment in which a coin was tossed thirty times. Find the experimental probability of tossing tails for this experiment.**



$$P(\text{tails}) = \frac{\text{number of times tails occurs}}{\text{number of possible outcomes}}$$

$$= \frac{18}{30} \text{ or } \frac{3}{5}$$

The experimental probability of tossing tails is $\frac{3}{5}$.

3 **Compare the experimental probability you found in Example 2 to its theoretical probability.**

The theoretical probability of tossing tails on a coin is $\frac{1}{2}$. So, the experimental probability is close to the theoretical probability.

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EXAMPLES Predict Future Events

4 **FOOD** **In a survey, 100 people were asked to name their favorite Independence Day side dishes. What is the experimental probability of macaroni salad being someone's favorite dish?**

Side Dish	Number of People
potato salad	55
green salad or vegetables	25
macaroni salad	12
coleslaw	8

There were 100 people surveyed and 12 chose macaroni salad. So, the experimental probability is $\frac{12}{100}$, or $\frac{3}{25}$.

5 **Suppose 250 people attend the city's Independence Day barbecue. How many can be expected to choose macaroni salad as their favorite side dish?**

$\frac{3}{25} = \frac{x}{250}$ Write a proposition.

$3 \cdot 250 = 25x$ Find the cross products.

$30 = x$ About 30 will choose macaroni salad.

Your Turn

a. What is the experimental probability of potato salad being someone's favorite dish?

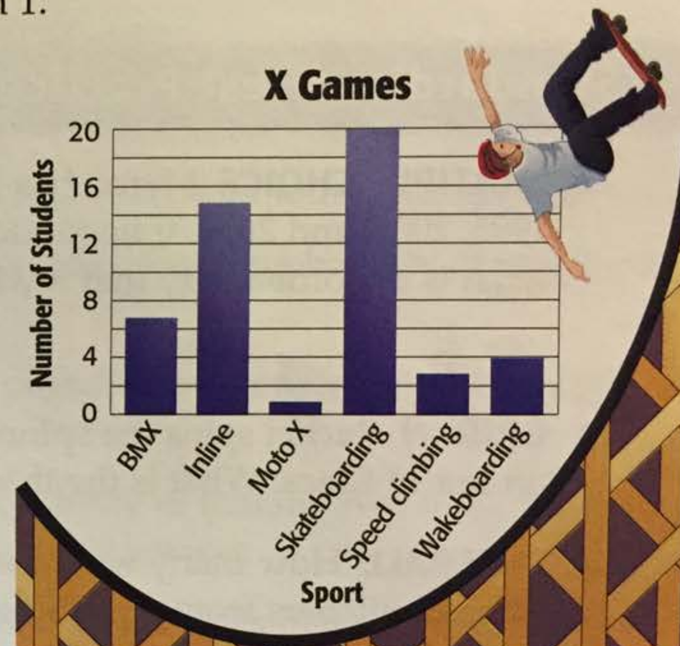
b. About how many people can be expected to choose potato salad as their favorite dish if 250 attend the barbecue?

For Exercises 6–9, a number cube is tossed 20 times and lands on 1 two times and on 5 four times.

HOMEWORK HELP	
For Exercises	See Examples
6–9	1–3
10, 15	4
11–14, 16–17	5
Extra Practice See pages 586, 604.	

6. Find the experimental probability of landing on 5.
7. Find the theoretical probability of *not* landing on 5.
8. Find the theoretical probability of landing on 1.
9. Find the experimental probability of *not* landing on 1.

X GAMES For Exercises 10–12, use the graph of a survey of 50 students asked to name their favorite X Game sport.



10. What is the probability of inline being someone's favorite sport?
11. Suppose 500 people attend the X Games. How many can be expected to choose inline as their favorite sport?
12. Suppose 500 people attend the X Games. How many can be expected to choose speed climbing as their favorite sport?
13. **REFRESHMENTS** In a survey taken at the beach, 47 people preferred cola, 28 preferred root beer, and 25 preferred ginger ale. If the manager of the Beach Hut is going to buy 50 cases of soda for the next day, about how many cases should be root beer?
14. **SPINNERS** A spinner marked with three sections A, B, and C was spun 100 times. The results are shown in the table. Make a drawing of the spinner based on its experimental probabilities.

Section	Frequency
A	24
B	50
C	26

In the Mini Lab, choosing the heat and the lane is a compound event. A **compound event** consists of two or more simple events. Since choosing the heat number does not affect choosing the lane number, both events are called **independent events**.

Key Concept — Probability of Two Independent Events

Words The probability of two independent events can be found by multiplying the probability of the first event by the probability of the second event.

Symbols $P(A \text{ and } B) = P(A) \cdot P(B)$

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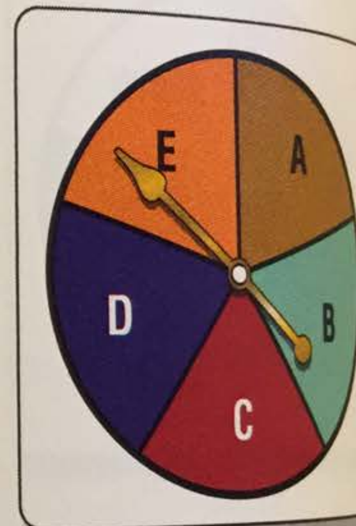
EXAMPLE **Independent Events**

1 **A number cube is rolled, and the spinner at the right is spun. Find the probability of rolling a 2 and spinning a vowel.**



$P(2) = \frac{1}{6}$ $P(\text{vowel}) = \frac{2}{5}$

$P(2 \text{ and vowel}) = \frac{1}{6} \cdot \frac{2}{5} \text{ or } \frac{1}{15}$

So, the probability of rolling a 2 and spinning a vowel is $\frac{1}{15}$.

Check You can make a tree diagram to check your answer.

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If the outcome of one event affects the outcome of a second event, the events are called **dependent events**.

EXAMPLE Dependent Events

2 **SNACK BARS** A box contains 2 oatmeal, 3 strawberry, and 6 cinnamon snack bars. Ruby reaches in the box and randomly takes two snack bars, one after the other. Find the probability that she will choose a cinnamon bar and then a strawberry bar.

$P(\text{cinnamon}) = \frac{6}{11}$ ← 11 snack bars, 6 are cinnamon

$P(\text{strawberry}) = \frac{3}{10}$ ← 10 snack bars after 1 cinnamon snack bar has been removed, 3 are strawberry

$$P(\text{cinnamon, then strawberry}) = \frac{\overset{3}{\cancel{6}}}{11} \cdot \frac{3}{\underset{5}{\cancel{10}}} \text{ or } \frac{9}{55}$$

So, the probability that Ruby will choose a cinnamon snack bar and then a strawberry snack bar is $\frac{9}{55}$, or about 16%.

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Key Concept — **Probability of Two Dependent Events**

Words The probability of two dependent events is the probability of the first event times the probability that the second event occurs after the first.

Symbols $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$

6. A red and a blue number cube are rolled. Find the probability that an odd number is rolled on the red cube and a number greater than 1 is rolled on the blue cube.

7. Find the probability of heads on three consecutive tosses of a coin.

8. A cooler is filled with 12 colas and 9 diet colas. If Victor randomly chooses two without replacing the first, what is the probability that he will choose a cola and then a diet cola?

9. A deck of 30 cards is made up of the numbers 1–10 in three colors: red, purple, and green. Two cards are selected without either being replaced. Find the probability of choosing a purple 5 and then a red or green card.

10. Draw a Venn diagram to show the probability of two independent events *A* and *B*.

11. **CIVICS** In the 108th Congress, Tennessee had 4 Republicans and 5 Democrats serving in the House of Representatives. If a subcommittee of 2 representatives was formed to study Internet usage among middle school students, what is the probability that both would be Republicans?

HOMEWORK HELP

For Exercises	See Examples
6–7, 17–19	1
8–9, 11–14	2

Extra Practice
See pages 586, 604.

CARDS For Exercises 12–14, use the information below.

A standard deck of playing cards contains 52 cards in four suits of 13 cards each. Two suits are red and two suits are black. Two cards are chosen from the deck one after another. Find each probability.

12. $P(2 \text{ hearts})$

13. $P(\text{red, black})$

14. $P(\text{Ace, King})$